# ELECTRIC FIELD INTENSITY

## Definitions

An **electric field** is a region where an electric force is experienced by a charge. Electric fields can be represented by electric field line.

An **electric field line** is the path taken by a small positive charge placed in the electric field.

## Properties of electric field lines

- They begin or end on charge.
- They are in a state of tension which causes them to shorten.
- They repel one another sideways
- They never cross each other
- The electric field  line are symmetrical on point charges
- The number of filed lines originating or terminating on a charge is proportional to the magnitude of the charge

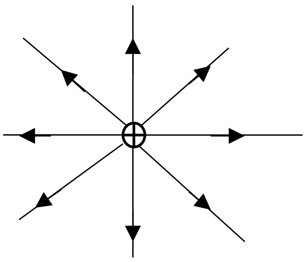
## Direction of electric field

The direction of the electric field at any point is the direction of force on a small positive test charge placed at that point.

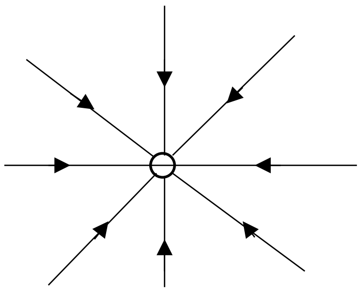
## Electric field Patterns

Electric field lines are used to represent the distribution and strength of the electric field around charges. The figures below show the electric patterns around some charge distribution.

1) Isolated point charge

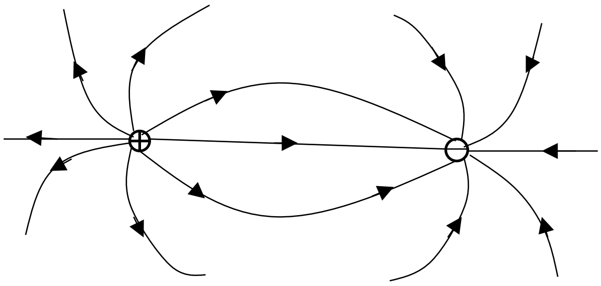


a) positive charge

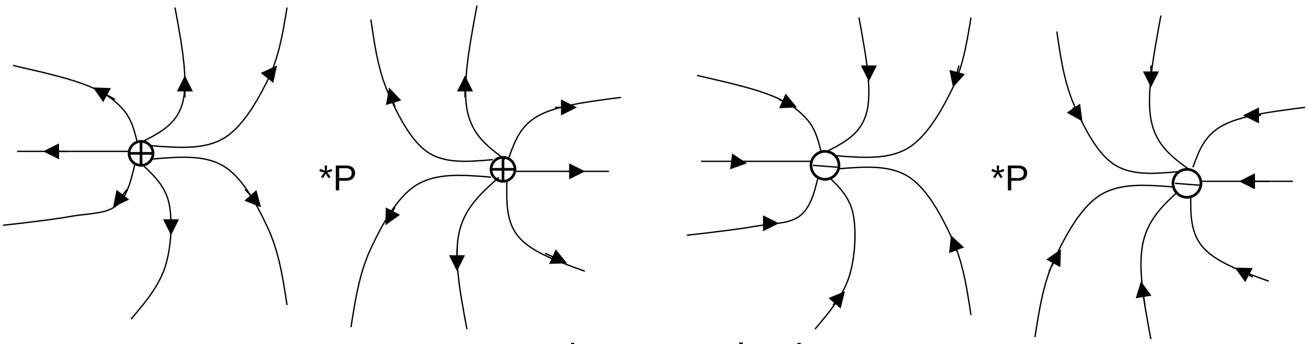


b) negative charge

2) Two point opposite charges close to each other



3) Two point charges of similar charges close to each other




* P is a neutral point

a) positive charges b) negative charges

A **neutral point** is defined as point in an electric field where there is no electric force experienced by a charge placed at that point.

4) Point charge close to a plate

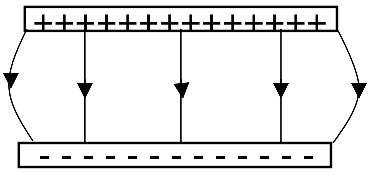
a) positive charge b) negative charge

5) Point charge close to a plate of similar charge


a) positive charge b) negative charge

*P is a neutral point

6) Parallel plates of opposite charge



7) Parallel plates of similar charge


*P is a neutral point

a) positive charge b) negative charge

**Why the electric field lines close to the conductor are always at right angles to the surface of the conductor.**

- Very close to the surface of a conductor, there is maximum force exerted on the charge; because the distance from the conductor is very small.
- The electric field experienced is always in the direction of the force and is little influenced by neighbouring electric fields.
- The maximum force (and hence the electric field) is directed along the normal to the surface. Therefore the electric field is normal to the surface of the conductor.

ELECTRIC FIELD INTENSITY

**Electric field intensity** is defined as the force experienced by a positive charge of 1C placed at a point in an electric field.

From Coulomb's law


The force between two point charges is given by

$$F = \frac{Q\,Q_o}{4\,\pi\varepsilon_o r^2} = K\frac{Q\,Q_o}{r^2} \qquad \text{.................................. (1)}$$

When $Q_o$ = +1C, then F=E

$$\text{Hence } E = \frac{Q}{4\,\pi\varepsilon_o r^2} = K\frac{Q}{r^2} \qquad \text{.................................. (2)}$$

Relation between Electric field intensity and electric Force

(1) ÷ (2) gives $\frac{F}{E} = Q_o$

$$F = E\,Q_o \qquad \text{.................................. (3)}$$

SI unit of electric field intensity:

Generally from eqn (3)

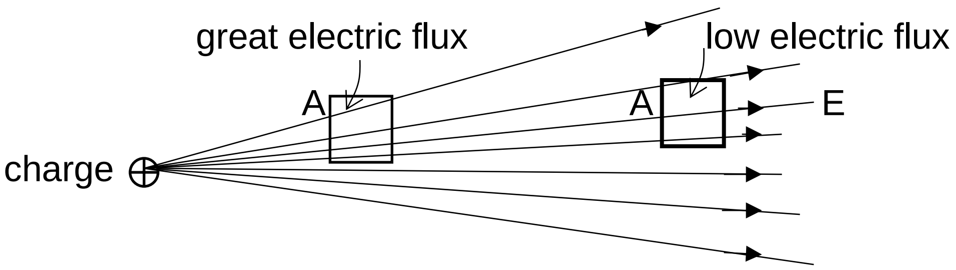
$$E = \frac{F\,(N)}{Q\,(C)}$$

Hence electric field intensity E is measured in newton per coulomb ($NC^{-1}$)

**NB**: Electric field intensity is a vector quantity and therefore has both magnitude and direction.

EXAMPLES.(*Already covered in class*)

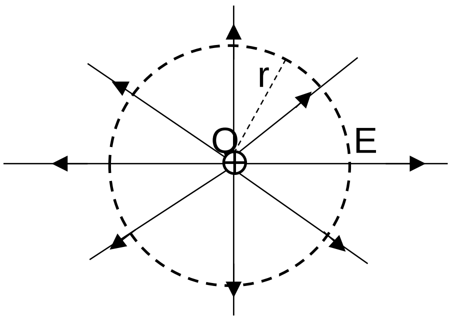
ELECTRIC FLUX Ø




This is the product of electric field strength at any point and area normal to the field
Ø=$AE$

TOTAL ELECTRIC FLUX (Guass's theorem)

Consider a spherical surface of radius r, concentric with point charge Q as shown in the figure below.




The electric field intensity on the spherical surface

$$E = \frac{Q}{4\,\pi\varepsilon_o r^2}$$

The area of the spherical surface is A = $4\pi r^2$

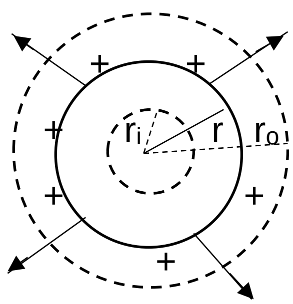
Electric flux Φ= EA

$$= \frac{Q}{4\,\pi\varepsilon_o r^2} \times 4\pi r^2$$

$$= \frac{Q}{\varepsilon_o} \text{ (Gauss' theorem)}$$

**Guass's theorem** of electrostatic states that the total electric flux passing normally through a closed surface, whatever its shape is given by the ratio of charge enclosed to the permittivity of the medium.

**Electric field intensity due to hollow charged sphere**



Consider a hollow sphere of radius r, carrying a charge Q as shown in the figure above.

(i) <u>inside the sphere</u>

Consider a spherical surface of radius $r_i$ drawn inside the sphere as shown in the figure above. The net charge inside a hollow sphere is Zero (as per Faradays ice pail experiment). Hence the electric field intensity E= $\frac{Q}{4\,\pi\varepsilon_o r^2}$ is zero everywhere inside the sphere.

(ii) <u>on the surface of the sphere</u>

The area of the surface along the sphere is given by A=$4\pi r^2$ and the charge enclosed by the surface of the sphere is Q.

From Gauss' theorem the product E x A = $\frac{Q}{\varepsilon_o}$

$$E \times 4\pi r^2 = \frac{Q}{\varepsilon_o}$$

The electric field intensity E= $\frac{Q}{4\,\pi\varepsilon_o r^2}$

(iii) outside the sphere

The electric flux through a spherical surface of radius $r_o$ drawn concentric with the sphere carrying a charge Q is given by

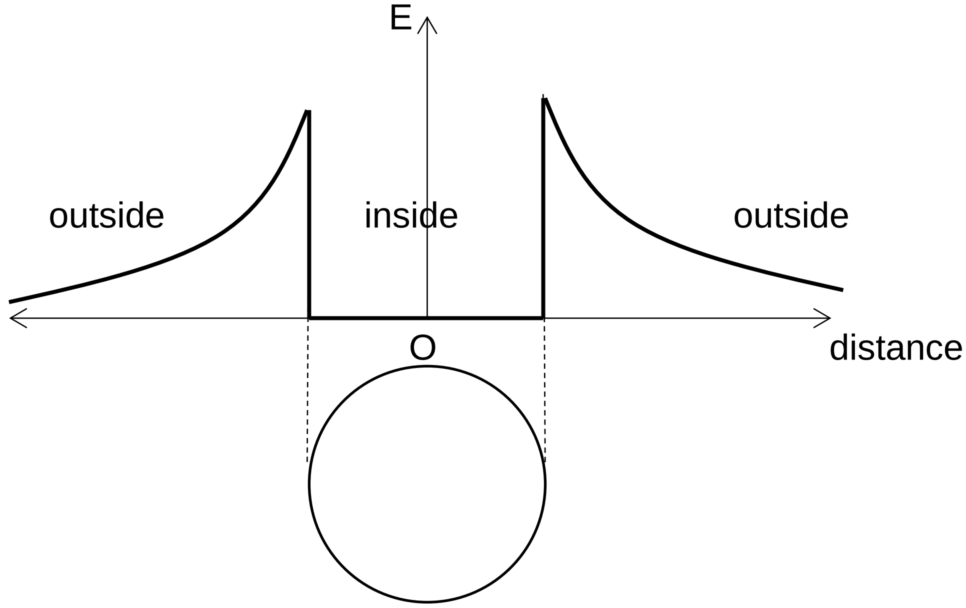
$$\text{Electric flux } E \times A = \frac{Q}{\varepsilon_o}$$

$$E \times 4\pi {r_o}^2 = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{4\,\pi\varepsilon_o {r_o}^2}$$

$$\text{Hence } E \propto \frac{1}{r^2}$$

The graph below shows the variation of electric field intensity E with the distance from a point on the far left to a point on the far right of the charged sphere.



ELECTRIC POTENTIAL

## Electric potential difference

This is the work done in moving a positive charge of 1C from one point to another.

## SI Unit:

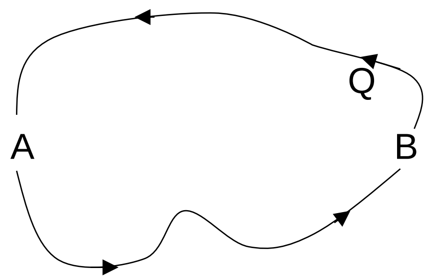
The SI unit of potential difference is a **volt.** A volt is defined as the potential difference between two points A and B in which 1J of energy is used to transfer one coulomb of positive charge from B to A.

## Electric potential and energy

From the definition of a volt, if a charge Q (coulombs) is moved through a p.d V(volts) the work done W (joules) is given by

$$W = QV$$

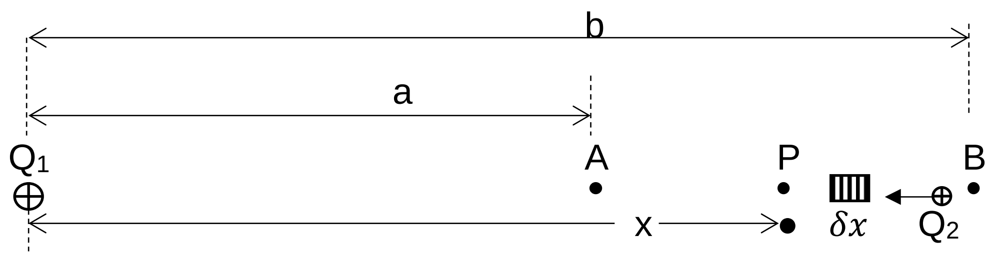
Consider two points A and B in an electric field in which A is at a higher potential than B.




If a positive charge Q is moved from B to A work is done on it of $V_{AB}Q$ and the charge gains this potential energy.

If the charge goes back from A to B it loses that potential energy and work is done on the charge by the electrostatic force.

## Expression for potential energy between charges




The electrostatic force on a charge $Q_2$ at a distance x from a charge $Q_1$ is given by

$$F = K\frac{Q_1 \, x \, Q_2}{x^2}$$

$$\text{Hence } F = K\frac{Q_1 Q_2}{x^2}$$

The work done by an external force in moving $Q_2$ through a distance $\delta x$ is given by

$$\delta w = -F\delta x = -K\frac{Q_1 Q_2}{x^2}\delta x$$

negative because work is done against the force of repulsion between the charges

The total work done in moving $Q_2$ form B to A at a distance b and a respectively from Q gives the electric potential energy gained

$$W = \int dw = -KQ_1Q_2\int_b^a \frac{1}{x^2}dx$$

$$W = -KQ_1Q_2\left[\frac{1}{-x}\right]_b^a = KQ_1Q_2\left[\frac{1}{x}\right]_b^a$$

$$\therefore\ W = KQ_1Q_2\left[\frac{1}{a}-\frac{1}{b}\right]$$

If the charge $Q_2$ is moved from infinity to a point P at a distance r from $Q_1$ then a=r and b=∞, the potential energy between the charges is given by

$$W = KQ_1Q_2\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

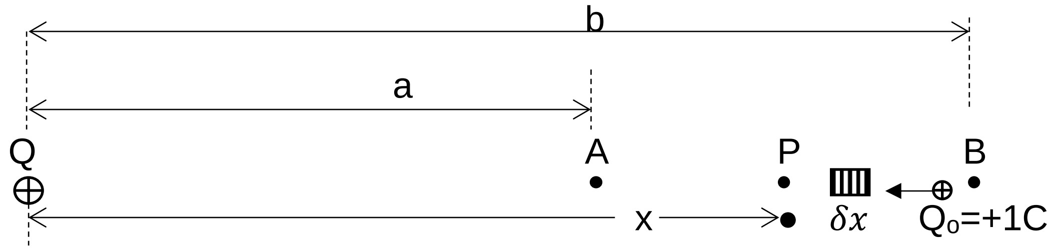
But $\frac{1}{\infty} \longrightarrow 0$

Hence potential energy $W = K\frac{Q_1Q_2}{r}$

**What happens to the potential energy as two point charges of the same sign are brought together**.

- Like charges repel. Work has to be done against the repulsive forces between them to bring them closer.
- This work is stored as electric potential energy of the system.
- The potential energy of the two like charges therefore increases when the charges are brought closer together

**Expression for potential difference**




The electrostatic force on a charge $Q_o$ at a distance x from a charge Q is given by

$$F = K\frac{Q\ x\ Q_o}{x^2} = K\frac{Q\ x\ 1}{x^2}$$

Hence $F = K\frac{Q}{x^2}$

The work done by an external force in moving $Q_o$ through a distance $\delta x$ is given by

$$\delta w = -F\delta x = -K\frac{Q}{x^2}\delta x$$

negative because work is done against the force of repulsion between the charges

The total work done in moving $Q_o$ (of positive 1C) form B to A at a distance b and a respectively from Q gives the electric potential difference $V_{AB}$ between A and B

$$V_{AB} = \int dw = -KQ\int_b^a \frac{1}{x^2}dx$$

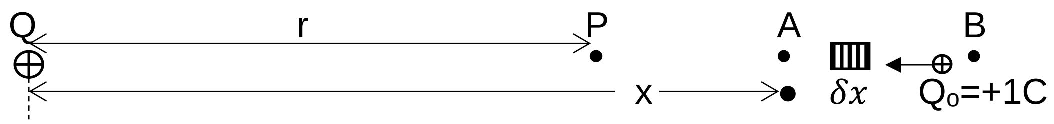
$$V_{AB} = -KQ\left[\frac{1}{-x}\right]_b^a = KQ\left[\frac{1}{x}\right]_b^a$$

$$\therefore\ V_{AB} = KQ\left[\frac{1}{a} - \frac{1}{b}\right]$$

**Electric potential at a point**

This is the work done in moving a positive charge of 1C from infinity to a point against the electric field.

**Expression for electric potential at a point**




The electrostatic force on a charge $Q_o$=+1C at a distance x from a charge Q is given by

$$F = K\frac{Q\,x\,Q_o}{x^2} = K\frac{Q\,x\,1}{x^2}$$

$$\text{Hence } F = K\frac{Q}{x^2}$$

The work done by an external force in moving $Q_o$ through a distance $\delta x$ is given by

$$\delta w = -F\delta x = -K\frac{Q}{x^2}\delta x$$

negative because work is done against the force of repulsion between the charges

The total work done in moving $Q_o$ (of positive 1C) from infinity to a point P at a distance r from Q gives the electric potential difference V at P

$$V = \int dw = -KQ\int_\infty^r \frac{1}{x^2}dx$$

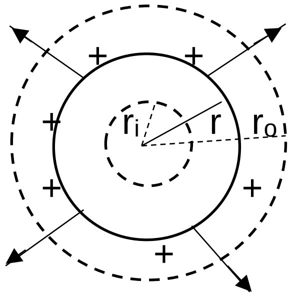
$$V = -KQ\left[\frac{1}{-x}\right]_\infty^r = KQ\left[\frac{1}{x}\right]_\infty^r$$

$$V = KQ\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

But $\frac{1}{\infty} \longrightarrow 0$

$$\therefore\ V = K\frac{Q}{r}$$

**Electric potential due to a hollow charged sphere**




Consider a hollow sphere of radius r, carrying a charge Q as shown in the figure above.

(i) inside the sphere

Consider a spherical surface of radius $r_i$ drawn inside the sphere as shown in the figure above. No net charge resides on the inside of a hollow conductor. Therefore there is no work done to transfer a charge from one point to another a point inside the sphere. All points inside and on the surface of the sphere have the same potential.

(ii) on the surface of the sphere

The charge Q lies on the surface of the sphere of radius r.

The electric potential $V = \frac{Q}{4\,\pi\varepsilon_o r}$

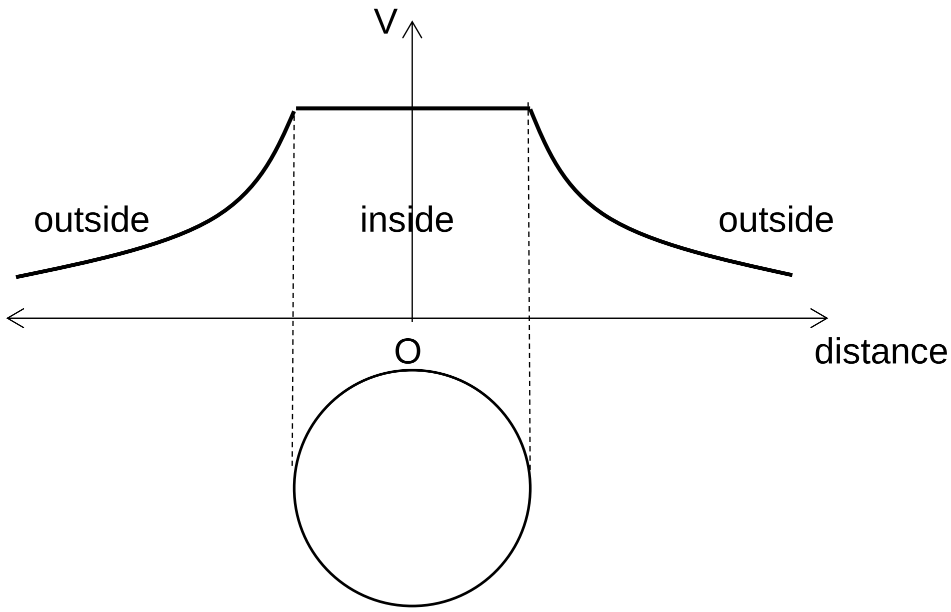
(iii) outside the sphere

The work done in moving a positive charge of 1C to a point a distance $r_o$ from the center of the sphere carrying a charge Q is given by

$$V = \frac{Q}{4\,\pi\varepsilon_o r_o}$$

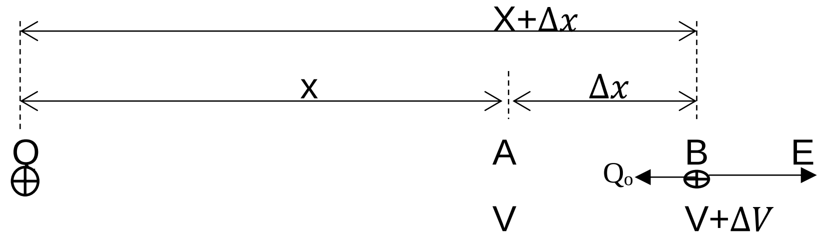
$$\text{Hence } V \propto \frac{1}{r}$$

The graph below shows the variation of electric potential V with the distance from a point on the far left to a point on the far right of the charged sphere.



ELECTRIC POTENTIAL AND ELECTRIC FIELD INTENSITY (Electric Potential Gradient)

Consider two points A and B separated by a distance $\Delta x$ in an electric field E as shown in the figure below:




If A and B are at electric potentials V and V +$\Delta V$ respectively, then

the potential difference between A and B

$$V_{AB} = V_A - V_B$$

$$= V - (V + \Delta V)$$

$$= -\Delta V \quad \text{.........................} \quad (1)$$

The work done in moving a positive charge $Q_o$ =+1C from B to A

$$\Delta w = \text{force x distance}$$

$$= F \times \Delta x$$

But $F = EQ_o$

$$\Delta w = EQ_o \Delta x$$

Since work done in moving a charge $Q_o$ =+1C gives the electric potential $V_{AB}$ between A and B then

$$V_{AB} = \frac{\Delta W}{Q_O} = E\Delta x \quad \text{..............................} \quad (2)$$

Using eqn (1) and (2)

$$E\Delta x = -\Delta V$$

$$E = \frac{-\Delta V}{\Delta x}$$

In the limit as $\Delta x \longrightarrow 0$

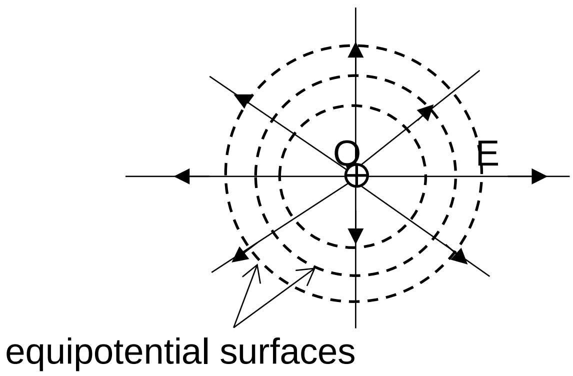
$\therefore E = -\frac{dV}{dx}$ the potential gradient

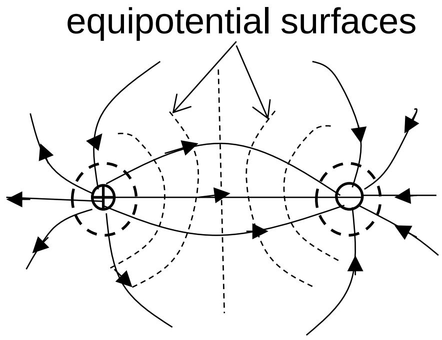
**Other Unit of Electric field strength E**

Since V is in volts and $x$ in meters then E =$\frac{dV}{dx}$ is in volts per meter ($Vm^{-1}$)

EQUIPOTENTIAL SURFACES

An equipotential surface is any surface over which the electric potential is constant.



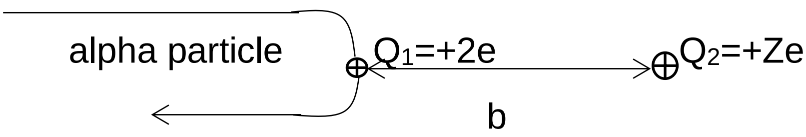

a) isolated point charge        b) two opposite point charges

**Properties of equipotential surface**

- The work done moving charge from one point on surface to another is zero.
- The electric field is always at right angles to equipotential surfaces.
- There is no electric field along the surface
- There is no potential gradient lying in the equipotential surface

Closest distance of approach

Consider alpha particles incident head-on to the nucleus of an atom in a foil.




At the closest distance of approach, all the kinetic energy of the alpha particles is converted into electrostatic potential energy of the alpha particle-nucleus system. If u is the initial speed of the alpha particles, then

$$\frac{1}{2}mu^2 = K\frac{Q_1Q_2}{r}$$

$$\frac{1}{2}mu^2 = \frac{(2e)(Ze)}{4\pi\varepsilon_o b}$$

The closest distance of approach b = $\frac{Ze^2}{\pi\varepsilon_o mu^2}$

EXAMPLES (*leave 2 pages space*)